# 3.2

## Exponential and Logistic Modeling

### Exponential Population Model

If a population *P* is changing at a constant rate *r* each year, then

$$P(t) = P_0(1+r)^t,$$

where $P_0$ is the initial population, *r* is expressed as a decimal, and *t* is time in years.

Note: If $(1 + r) > 1$ or when $r > 0$ then, it is growth. If $(1 + r) < 1$ or when $r < 0$ then it is decay.

### Finding Growth and Decay Rates

Tell whether the population model is an exponential growth function or exponential decay function, and find the constant percentage rate of growth or decay (*r*).

a) $P(t) = 782{,}248 \cdot 1.0136^t$

Because $1 + r = 1.0136$, $r = .0136 > 0$. So, *P* is an exponential growth function with a growth rate of 1.36%.

b) $P(t) = 1{,}203{,}368 \cdot 0.9858^t$

Because $1 + r = 0.9858$, $r = -0.0142 < 0$. So, *P* is an exponential decay function with a decay rate of 1.42%.

### Finding an Exponential Function

Determine the exponential function with initial value = 5, increasing at a rate of 4% per year.

Because $P_0 = 5$ and $r = 4\% = 0.04$, the function is $P(t) = 5(1 + 0.04)^t$, or $P(t) = 5 \cdot 1.04^t$.

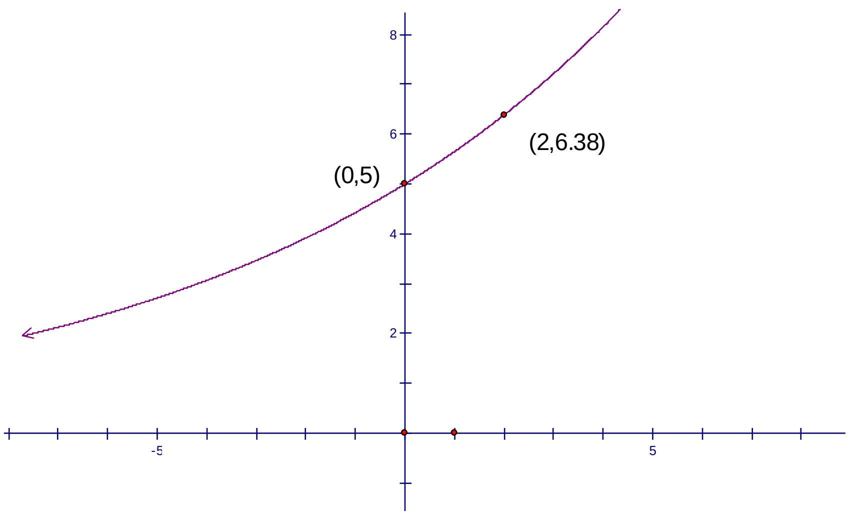
Determine the exponential function with initial value = 123, decreasing at a rate of 5.6% per year.

Because $P_0 = 123$ and $r = -5.6\% = -0.056$ (decreasing), the function is $P(t) = 123(1 - 0.056)^t$, or $P(t) = 123 \cdot 0.944^t$.

## Using Regression to Model Population

See example 6, pg. 293.

## Using a graph to determine an Exponential Formula




Using $f(x) = f_0 \cdot b^x$, $f_0 = 5$, so $f(x) = 5 \cdot b^x$. Using the point (2, 6.38) we get

$6.38 = 5 \cdot b^2$

Solving for b we get $6.38/5 = b^2$ , so $b = 1.12$.

The equation for the exponential function for the graph is $f(x) = 5 \cdot 1.12^x$.

## Finding Logistic Functions

Recalling from 3.1 the logistic models:

$$f(x) = \frac{c}{1 + a \bullet b^x} \quad or \quad f(x) = \frac{c}{1 + a \bullet e^{-kx}} \ .$$

Example:

Find the logistic function that satisfies the given conditions.

Initial value = 3, limit to growth = 32, passing through (1, 15).

Using the model $f(x)=\dfrac{c}{1+a\bullet b^x}$ and the given information, we can solve for b.

$$3=\frac{32}{1+a\bullet b^0} \quad\rightarrow\quad 3=\frac{32}{1+a} \quad\rightarrow\quad 3(1+a)=32 \quad\rightarrow\quad 1+a=\frac{32}{3} \quad\rightarrow\quad a=\frac{29}{3}$$

$$3=\frac{32}{1+a}$$

$$a=\frac{29}{3}$$

$15=\dfrac{32}{1+\frac{29}{3}\bullet b^1}$ → (1 + 29/3b)15 = 32 → b = 17/145 so the equation is $f(x)=\dfrac{32}{1+\frac{29}{3}\bullet\left(\frac{17}{145}\right)^x}$

**Other Formulas you will need for this section:**

$$P(t)=P_0 2^{\frac{t}{k}} \quad double$$

$$P(t)=P_0\left(\frac{1}{2}\right)^{\frac{t}{k}} \quad half-life$$

$$y=a\bullet b^x$$

where b is the growth factor , a is the initial value.